CENTRAL LIMIT THEOREM Worksheet: NAME Last_____First_____ Class Time:____

Objective: Students will use probability and the Central Limit Theorem to analyze and improve the building safety requirements for the maximum load and occupancy of an observation deck.

1. An observation deck is being built as a tourist attraction in a city. The city building code states that to support 50 people, its maximum weight capacity should be 9000 pounds. A capacity of 9000 pounds and 50 people is equivalent to an average weight of 180 pounds per person: 9000 pounds ÷ 50 people = 180

   The United States periodically conducts a national Health and Nutrition Examination Survey (NHANES). Data from this survey shows that as of 2002:

   the average weight for all adult women in the US is approximately 160 pounds
   the average weight for all adult men is approximately 190 pounds.

   For a mixed group gender of men and women, assume **the average weight for all adults is 175 pounds**. We will **use 40 pounds as the standard deviation for the weight of an individual** in each of these categories of groups.

X = (description in words)_____

$\overline{X}$ = (description in words)_____

If the weight is capacity is exceeded and the deck is overloaded, the deck might collapse and cause serious injury or even death. There would also be financial losses if it collapses.

Find the probability that the deck's weight capacity will be overloaded, for each group below.
Do this by finding the probability that the average weight of the people in the group is more than the average permitted weight. In this case the average permitted weight is 9000 pounds ÷ 50 people = 180

1a. For a sample of **50 women**, find the probability that the average weight is more than 180 pounds.

$\overline{X}$ ~_____ Find $P(\overline{X} > 180)$: $P(\overline{X} > 180)$ = _____ (fill answer in the blank)
Show your calculator command. Draw the graph.

1b. For a sample of **50 men**, find the probability that the average weight is more than 180 pounds.

$\overline{X}$ ~_____ Find $P(\overline{X} > 180)$: $P(\overline{X} > 180)$ = _____ (fill answer in the blank)
Show your calculator command. Draw the graph.

1c. For a **mixed gender group of 50 people, (men and women)**, find the probability that the average weight is more than 180 pounds. (Hint: Use the average adult weight of 175 pounds.)

$\overline{X}$ ~_____ Find $P(\overline{X} > 180)$: $P(\overline{X} > 180)$ = _____ (fill answer in the blank)
Show your calculator command. Draw the graph.

1d. On next page, use the calculations above to draw conclusions about whether the deck is safe or unsafe.

1d. Would you feel safe with the maximum load recommendations for this observation platform, based on the probabilities above?

Circle your answers *AND* in one to three complete sentences, briefly explain, using probabilities to justify your conclusion. *You are graded on your conclusions AND the quality of your explanation so write carefully.*

Safe for Women: Yes No	Safe for Men: Yes No	Safe for Mixed Gender Group: Yes No

When you finish this page, go to the next page for questions 2 and 3.

2. In question 1, you've identified some situations in which the deck is not safe – the chance of overloading the deck's weight limit is too large to be an acceptable risk. We need to make the deck safer.
We can make the deck safer by fixing and reinforcing the deck to make it stronger (expensive , time-consuming) or we can do that by limiting the number of people permitted on the deck (faster, cheaper).

Suppose you want to increase safety by **reducing** the number of people permitted on the observation deck to permit only 45 people. That permits an average weight of 200 pounds: 9000 pounds ÷ 45 people = 200
Use the same data from the National Health and Nutrition Examination Survey:

The average weight for all adult women is 160 pounds; average weight for all adult men is 190 pounds. For a mixed group of men and women, assume that the average weight for all adults is 175 pounds. Use 40 pounds as the standard deviation for the weight of an individual.

$\overline{X}$ = (description in words)_____

2a. For a sample of **45 women** , find the probability that the average weight is more than 200 pounds.

$\overline{X}$ ~_____ Find P($\overline{X}$ >200): P($\overline{X}$ >200) = _____ (fill answer in the blank)
Show your calculator command. Graph optional but recommended.

2b. For a sample of **45 men**, find the probability that the average weight is more than 200 pounds.

$\overline{X}$ ~_____ Find P($\overline{X}$ >200): P($\overline{X}$ >200) = _____ (fill answer in the blank)
Show your calculator command. Graph optional but recommended.

2c. For a **mixed gender sample of 45 people**, find the probability that the average weight is more than 200 pounds.

$\overline{X}$ ~_____ Find P($\overline{X}$ >200): P($\overline{X}$ >200) = _____ (fill answer in the blank)
Show your calculator command. Graph optional but recommended.

2d. Would you feel safe with a maximum load of 9000 pounds and 45 people for this observation platform, based on the probabilities above? Circle your answers *AND* in one to three complete sentences, briefly explain, using probabilities to justify your conclusion.
You are graded on your conclusions AND the quality of your explanation so write carefully.

Safe for Women: Yes No	Safe for Men: Yes No	Safe for Mixed Gender Group: Yes No

_____
_____
_____
_____
_____

3. DO FOR RANDOM SAMPLES OF **MEN ONLY**,

since the deck is most risky for groups consisting of all men.

Calculate the probability of exceeding the weight capacity for various sample sizes
n = 50, 45, 44, 42, 40.
Then decide if it is safe or not.

- Your decisions are data-driven. You are making decisions based on probabilities.
- Some (not all) of the calculations below are identical to some calculations questions 1 and 2.
- Therefore your decisions about safety in those situations should be the same here as you decided in questions 1 and 2.
- If your decisions in questions 1 and 2 are not consistent with your decisions in question 3, review your decisions to develop consistent decisions based on the probability calculations.
-

To decide about safety, you may want to consider such things as:

- Which capacities would make you feel safe/unsafe if you were a male going on this deck?
- Which capacities seem safe/unsafe to let a male friend or family member go on this deck?
- As the owner's insurance company, which capacities would make you feel safe insuring this deck?

Capacity (sample size) n	Information needed to do the calculation: Maximum Average Weight $\left(\frac{9000}{n}\right)$	Population Mean Weight for Men μ	Appropriate Standard Deviation $\frac{\sigma}{\sqrt{n}}$	Calculate the **Value** for the Probability of Deck Failure $P\left(\overline{X} > \frac{9000}{n}\right)$=?	Conclusion based on probability of overloading the deck: Safe or Not Safe?
50					
45					
44					
42					
40					

Compare conclusions with a classmate:

What probabilities do you consider risky for overloading the weight limit of the deck and what probabilities you consider safe. What probabilities do your classmates consider risky vs safe? Do you and your classmates have the same personal criteria for safety?

Write a few sentences about your criteria for probabilities that you consider safe and how it compares to that of your classmates.

_____

_____

_____

_____

_____